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GROWTH IN STUDENT MATHEMATICAL UNDERSTANDING THROUGH PRECALCULUS STUDENT AND TEACHER INTERACTIONS

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Abstract: This paper investigates the role of teacher interaction in the development of mathematical understanding of five students who worked together on a math-modeling task. The dialogue between the teacher/researcher and students is analyzed. Preliminary findings suggest that where the mathematical thinking of the students was understood, interventions helped develop students' thinking.

Introduction

The students, engaged in conversation with the teacher, often give explanations for their ideas. A question arises as to what influence, if any, the teacher's response to those explanations have on student progress. This report examines dialogue between teacher and students and seeks to investigate the effect on students' growth in mathematical understanding.

The data come from a two-week summer institute that was a component of a longitudinal study on the development of proof making in students'. The students worked in groups on precalculus level mathematics problems. This paper focuses on one group of students and one of the problems they examined.

Theoretical Framework

Communication is an essential part of the mathematics classroom. Communication provides a means for students to express their ideas and explain their thinking (NCTM, 2000). Through communication students can share ideas and discoveries about the mathematics on which they are working. The communication process helps students create meaning for their ideas. NCTM (2000) includes communication as one of the standards in *Principles and Standards for School Mathematics*. Because of the need for communication in the classroom, an environment can be created where teachers and students engage in very important dialogue. Dialogue is important because it helps teachers assess the student's mathematical understanding and allows the students to clarify and express their ideas. Towers (1998) developed several themes to describe teacher interaction with students and illustrated how these interactions occasion the growth of students' understanding. A teacher should be skilled in interacting with students in order to gain access to students' mathematical understanding. Teacher questioning can help students justify and extend ideas, make connections, and generalize their conjectures (Dann, Pantozzi, & Steencken, 1995). The development of these skills is not immediate for the teacher, but once gained the teacher has an effective way to facilitate the growth of a student's understanding (Martino & Maher, 1999).

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Being a participant in the classroom discourse, the teacher has an important function. In describing a classroom where students are working in small groups on a task, Maher, Davis, and Alston (1991) indicate that the teacher plays many roles: listening to children, offering suggestions, asking questions, facilitating discussions, drawing out justifications. When students discuss with their teachers the meaning of mathematical notions, students are expected to think about concepts, their meanings and their interrelations (Vinner, 1997). If students do think about concepts, they are in a conceptual mode of thinking (Vinner, 1997). If students do not think conceptually, but still produce answers which seem to be conceptual, then Vinner (1997) states the students are in a pseudo-conceptual mode of thinking. The teacher must continuously assess whether or not the students have learned the mathematical concept, truly understands the reasoning behind their problem solving approach, and can adequately support and defend their conclusions using their previously learned mathematical knowledge. In a regular classroom, it is not always possible to observe what a student does after an interaction with the teacher. Because this observation is not always possible, it is difficult for a teacher to determine if the interaction was beneficial to the student. Videotape data that follows the student when the teacher leaves make possible gaining a better understanding of a student's actions. Interacting with students is a challenging task for a teacher, who has to make instantaneous decisions. The researcher, who has the benefit of studying and referring to videotape data, however, can learn from the interaction after the fact. What the researcher learns from the interaction can be shared with the teachers, who can reflect on their actions, and facilitate a growth in students understanding.

Methodology

Participants

Five students seated at the same table (four males and one female) and one teacher/researcher were subjects in this study. All of the students were entering their fourth year of high school. The teacher/researcher involved in the interaction is an experienced professor of mathematics and mathematics education at the university level.

Task

The students were given a picture of a fossilized shell called *Placentiaceras*. The first part of the task, designed in 1991 by Robert Speiser, was to draw a ray from the center of the shell in any direction. Then with polar coordinates as a way to describe the spiral of the shell, the students were to make a table of r as a function of θ . After creating the table, the students were asked what they could say about r as a function of θ . The students had graphing calculators, transparencies, rulers, and markers at their disposal for completing this task.

Data Collection and Analysis

The data come from a two-hour videotape session during the third day of a two-week Institute. The interactions were coded to consider perspectives of the teacher and the students. For the students, the following codes were developed and used. S(i): Student ignores the suggestion made by the teacher; S(c): Student asks the teacher for clarification of a statement; S(a): Student attempts the teacher's idea or suggestion; S(e): Student engages in conversation for the purpose of explaining their own views. For the teacher: T(r): Teacher restates the problem or returns to an old idea; T(f): Teacher follows the student's idea or suggestion; T(n): Teacher introduces a new idea; T(c): Teacher asks the student to clarify their statements or idea. The codes were used to follow the choices of the teacher and the resulting action by the student. When students became engaged in a conversation, their words were examined for evidence of their understanding.

Findings

The students' own words demonstrate where mathematical understanding occurs, and where their growth in a solution to this problem appears. The teachers' insistence on reiterating previous ideas, as well as, moving on to new ideas and following the student's suggestions, allows for the opportunity for the students to advance their understanding. For example, a lack of understanding about their fourth power regression solution modeled using the TI-89 was observed when both students agreed that their model is a parabola.

- 00:52:35:17 E S(e) I think it does. I mean if you look, if you look at the regression. It's just like a parabola. And uh your data.
- 00:52:42:19 Mi It is a parabola
- 00:52:43:12 Ma S(e) It is a parabola. A very nice parabola And like you know. I mean you can't use anything behind past zero on the x obviously because it can't have negative growth. That doesn't make sense. So you can't do that. But I mean the way, the way it goes up and the reason why it goes sharply up is just the fact that. I mean even from here to here like say the distance is 6 then all of a sudden it is 40. It's not going to keep on going little by little. Eventually it's getting wider like this. And that's why it's jumping so high up. It's not the fact that it's off or it's not predicting anything. It's just the numbers are getting larger and larger. It has to go higher and higher. So that's why it goes that steep angle like that.

00:53:32:16 B T(f) Ok well. I am still interested in that earlier part because. Are you saying that this animal really started growing where we're saying theta equals zero is.

00:53:46:10 Ma Um hmm

00:53:48:21 B How do you know that

00:53:51:02 Ma S(e) It's gotta start somewhere. And it doesn't start. You can't start with. You can't start.

00:53:55:24 Mi You can't start with anything negative

00:53:57:24 Ma anything past nothing.

00:53:57:24 Mi Yeah

00:53:59:09 Ma You know

00:54:01:05 Mi Cause then it doesn't exist. In which case it's not there.

The students do not look further into the data beyond a visual fit of a scatter plot and their curve. The teacher/researcher listens the discussion about the model by focusing on where the shell started growing. The teacher/researcher returns to the idea about how the model describes the start of the growth of the shell.

00:57:20:24 B T(r) See then I am wondering about that fourth power model cause if you go to the left on it. You are sort of going inward on the shell right. You are going backward in time.

00:57:32:04 Ma Yeah

00:57:33:05 B But then suddenly as you keep going left it goes up.

00:57:37:10 Ma S(e) Oh but there is nothing there though. That is the thing. Like you have to set limitations somewhere because some things are just physically impossible you know.

00:57:45:24 B I think we're beginning to understand each other.

00:57:48:02 Ma Yeah

00:57:48:19 B Okay, Umm

00:57:51:09 Ma S(e) I mean its like. I guess its like certain things like if you figure out like differences with like electricity or something or like in physics. Like you can't have things that are. Sometimes you can't have things that are negative. There are things that are just physically impossible to have. And that to have something, to have an animal or a living thing that is a negative distance would mean that it isn't there. So it's not physically possible to have that anything past that zero. You know. It just wouldn't be there. This animal would not be there if there was a negative number. Basically.

The teacher/researcher continues to question the students. He asks the students to clarify their ideas in order to allow them to provide evidence for building their understanding of why their model does not work for certain values.

- 01:01:47:12 B T(c) Oh, so there's a place. Okay then you are agreeing that there's a place where the regression doesn't model the animal
- 01:01:48:01 Mi S(e) You can put it so that the restriction has to be greater than zero.
- 01:01:52:25 Ma S(e) Yes, but that's necessary for other things too. There's limitations.
- 01:01:56:10 B Okay, Okay.
- 01:01:57:13 Ma S(e) like like the first graph we did with uh with the running thing, with the uh, with the thing you had to put limitations on it cause there were certain things that went past a certain time.

The teacher/researcher and the students continue the discussion by focusing on the accuracy of the model outside the range of their collected data. The question of what a model would look like if the data were collected again moves the conversation topic to the model's general shape. After this discussion, the teacher/researcher returned to the left side of the student's model. By the left and right side, the teacher/researcher and students are using the origin of the coordinate plane as their reference point. Therefore the left side would refer to negative values of time, and the right side would refer to positive values of time. When the teacher/researcher returned to the left side of the student model, the teacher/researcher and students revisited discussing the model's accuracy during negative values of time.

- 01:06:37:25 B T(r) So it looks like we are making sense on the right and then we got questions on the left. Is that fair.
- 01:06:44:25 Ma Sure, why not.
- 01:06:46:10 B Okay
- 01:06:46:18 Mi S(c) What possible questions could have on the left. It's dead. It doesn't exist.
- 01:06:51:02 B Well I just don't.
- 01:06:51:18 Ma S(e) Not even that. It's not even born yet
- 01:06:51:19 B T(r) It's very hard for me, yeah. It's very hard for me to believe that this at some point in the distance past
- 01:06:57:05 Mi It doesn't exist
- 01:06:57:29 B T(r) That that it was very large as the fourth power, as that fourth power curve suggests.

- 01:07:04:08 Mi S(e) Alright fine, we'll do a third power curve, it'll be very small. It'll be gone.
- 01:07:07:14 Ma S(e) No, you know what. You know what you
S(a) got to do. You set a we do that?

The students have provided a way to adjust the model so that it does not show a large shell when time is negative. Though the two students believe the left side of this model does not accurately portray the growth of the shell, their methods of correcting the inaccuracy are different. Mi wants to change the regression curve to the third power model, which would result in a new equation that models a different rate of change, and continues the inaccuracy of the model before the shell began to grow. Ma's explanation shows that he wants to remove the left side, but believes that the right side correctly models the growth of the shell.

Conclusions

The students' understanding of their model grew because they have provided justification for why the model should not represent the shell before it started to grow. However the students' understanding of the rate at which the shell is growing did not grow during this interaction. The dialogue showed the students used a fourth power regression to create a solution to the task. In the discussion, three students used the word parabola to describe the curve. Their early classification of the graph as parabolic demonstrated a limited understanding of the rate at which the shell grew. This is because parabolic and quartic curves represent different rates of growth. Later when Mi and Ma recommended changes for their model, they provided different methods for a correction. Mi suggested changing their regression to a third power, and Ma suggested restricting the left side of the model. Since Mi's correction used a different regression model, Mi did not make a connection between the rate of growth and the type of curve needed to model that growth. Neither of the students provided evidence as to why the model is quartic. E stated, "I mean if you look, if you look at the regression. It's just like a parabola." Mi and Ma both followed with "It is a parabola." Mi and Ma accepted the visual interpretation of the model by E. The students' earlier understanding about rate of growth did not grow during this interaction because they have not provided justification for the model's shape beyond the visual inspection.

Despite this misunderstanding, the teacher/researcher did not correct or criticize their comments. Rather the focus of the teacher/researcher was to discuss the start of the growth of the shell using the students' model. The students' explanation showed an understanding about their model around zero. When the teacher/researcher returned to the growth of the shell around zero, the students' level of understanding increased through engagement in the conversation. The student connected the limitations on the model to other physical situations. This showed a growth in understanding by providing a justification for why the limitation exists. However, the student still did

not give an explanation for how to adjust the model, which provided room for further growth. Despite a growth in understanding about how the shell is growing, the teacher/researcher asked the students to clarify their ideas. The two students expressed awareness that their model has limitations. When Ma referred to "the running thing" he drew on prior experience of why limitations are needed and therefore provides a mathematical grounding for his reasoning. The student referred to an earlier problem from this workshop, which has more meaning because it directly involved the student. This connection to another physical representation is more powerful than the representations mentioned earlier. The teacher/researcher returned to the idea of the left side of the model after a discussion of the general shape of the model. During this engagement, the students explained how to change the model so the left side did not exist. Mi stated, "It doesn't exist", but suggested the group use a third power regression to fix their model. Ma suggested, "You set a limitation on the graph so there is no left side and then we won't have this problem". The students demonstrated another growth in understanding by providing a method to correct the model. Previously the students explained why the shell could not exist for negative values, but have now moved forward to provide possible methods for representing the limitation on the model. Both of the students agreed the model does not accurately portray the growth of the shell by suggesting methods to alter their model. However, their understanding of the reasoning behind the inaccuracy differed which resulted in multiple methods for correcting the model.

Through interaction between the teacher and students, the students made public their level of mathematical understanding. By examining the episodes presented, one can see that the teacher/researcher consistently returned to the idea of how the model demonstrated the growth of the shell over the entire domain of the students' model. Additionally, the students' ideas are followed or they are asked to clarify their statements. Using this method of questioning, the students were given the chance to make connections and reorganize their thoughts in order to provide justification for their conclusions. In providing this justification, their understanding grew because the students suggested bases for their conclusions. By using previous knowledge as a method of justification, the students connected the current problem with other situations and showed a growth in understanding. The opportunity for growth occurred because the teacher continually returned to old ideas. As a consequence, the students had multiple possibilities to become engaged in conversations and articulate their understanding of the mathematics.

This research provides a foundation for continuing a dialogue about the affects of teacher and student interactions in the classroom. These preliminary findings imply that teacher interaction helps the student to express their mathematical understanding. Further research can help to indicate whether the teachers/researchers can learn from their choices during interactions to see if they are constructively contributing to students' progress. More research is needed to provide a better understanding of how

teacher intervention, particularly questioning, can contribute to students' mathematical understanding.

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